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Erratum

Remarks on Causality in Relativistic Quantum Field Theory

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It is shown that the correlations predicted by relativistic quantum field theory in locally normal states between projections in local von Neumann algebras $\mathcal{A}(V_1)$, $\mathcal{A}(V_2)$ associated with spacelike separated spacetime regions V_1, V_2 have a (Reichenbachian) common cause located in the union of the backward light cones of V_1 and V_2. Further comments on causality and independence in quantum field theory are made.

KEY WORDS: algebraic quantum field theory; causality; Reichenbach's Common Cause Principle.

1. INTRODUCTION

Algebraic quantum field theory (AQFT) (cf. Haag, 1992) predicts correlations between projections A, B lying in von Neumann algebras $\mathcal{A}(V_1)$, $\mathcal{A}(V_2)$ associated with spacelike separated spacetime regions V_1, V_2. According to *Reichenbach's Common Cause Principle* (cf. Salmon, 1984) if two events A and B are correlated, then the correlation between A and B is either due to a direct causal influence connecting A and B, or there is a third event C which is a common cause of the correlation. The latter means that C satisfies four simple probabilistic conditions which together imply the correlation in question.

The correlations predicted by AQFT lead naturally to the question of the status of Reichenbach's Common Cause Principle within AQFT. If the correlated projections belong to algebras associated with spacelike separated regions, a direct causal influence between them is excluded by the theory of relativity. Consequently, compliance of AQFT with Reichenbach's Common Cause Principle

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would mean that for every correlation between projections A and B lying in von Neumann algebras associated with spacelike separated spacetime regions V_1, V_2, there must exist a projection C possessing the probabilistic properties which qualify it to be a Reichenbachian common cause of the correlation between A and B. However, since observables and hence also the projections in AQFT must be localized, one also has to specify the spacetime region V with which the von Neumann algebra $\mathcal{A}(V)$ containing the common cause C is associated. Intuitively, the region V should be disjoint from both V_1 and V_2 but should not be causally disjoint from them, in order to leave room for a causal effect of C on the correlated events. There are three natural candidates for such a region V: the intersection of the backward light cones of V_1 and V_2 ($cpast(V_1, V_2)$, see (10)), the intersection of the backward light cones of every point in V_1 and V_2 ($spast(V_1, V_2)$, see (11)) and the union of the backward light cones of V_1 and V_2 ($wpast(V_1, V_2)$, see (9)). The requirement that the common cause belongs to local algebras associated with spacetime regions $spast(V_1, V_2)$, $cpast(V_1, V_2)$ and $wpast(V_1, V_2)$, leads to three different specifications of Reichenbach's Common Cause Principle in AQFT, called Strong Common Cause Principle, Common Cause Principle and Weak Common Cause Principle, respectively (Definition 3). Since $spast(V_1, V_2) = \emptyset$ if V_1 and V_2 are complementary wedge regions and AQFT predicts correlations between projections localized in complementary wedges (see below), the Strong Common Cause Principle fails in AQFT. Whether the Common Cause Principle holds is still an open problem.

We show that the Weak Common Cause Principle holds for every local system $(\mathcal{A}(V_1), \mathcal{A}(V_2), \phi)$ with a locally normal and locally faithful state ϕ and suitable, bounded spacelike separated spacetime regions V_1, V_2, if a net $\{\mathcal{A}(V)\}$ satisfies some standard, physically natural assumptions as well as the so–called *local primitive causality* condition (Definition 1). Such states include the states of physical interest in vacuum representations for relativistic quantum field theories on Minkowski space. We shall interpret our main result, Proposition 3, as a clear demonstration that AQFT is a causally rich enough theory to comply with the Weak Common Cause Principle – and possibly also with the Common Cause Principle.

In the next section we shall specify the assumptions and some immediate consequences of these assumptions needed in the proof of the main result. In Section 3 the definitions of the Reichenbach's Common Cause Principles for AQFT are given, followed by the main result. In the last section we shall make some further comments about our results.

2. SPACELIKE CORRELATIONS IN QUANTUM FIELD THEORY

Throughout the paper $\{\mathcal{A}(V)\}$ denotes a net of local von Neumann algebras (indexed by the open, bounded subsets V of Minkowski space M) satisfying the standard axioms of (i) isotony, (ii) Einstein causality, (iii) relativistic covariance and acting on a Hilbert space $\mathcal{H}$ carrying an irreducible vacuum representation of the net. The representation of the Poincaré group is therefore (iv) implemented by a (strongly continuous) unitary representation U satisfying the spectrum condition and having a distinguished invariant vector $\Omega \in \mathcal{H}$ representing the vacuum state. In addition to (i)–(iv), we also assume (v) weak additivity: for any nonempty open region V, the set of operators $\cup_{x\in\mathbb{R}^4}\mathcal{A}(V+x)$ is dense in $\cup_{V\subset M}\mathcal{A}(V)$ (in the weak operator topology). (For further discussion of these axioms, see Haag (1992) and Horuzhy (1990).)

An immediate consequence of assumptions (i)–(v) is that we may employ the following result of Borchers:

Proposition 1. (Borchers, 1965) *Under the assumptions (i)–(v), for any nonempty open region V, the set of vectors $\mathcal{A}(V)\Phi$ is dense in $\mathcal{H}$, for any vector Φ which is analytic for the energy.*

Note that any vector Φ with finite energy content, in particular the vacuum, is analytic for the energy. And since no preparation of a quantum system which can be carried out by man can require infinite energy, it is evident that (convex combinations of) states induced by such analytic vectors include all of the physically interesting states in this representation.

Note further that assumption (ii) entails that such vectors are also separating (*i.e.* $X \in \mathcal{A}(V)$ and $X\Phi = 0$ imply $X = 0$) for all algebras $\mathcal{A}(V)$ such that V' is nonempty. (Here V' denotes the causal complement and $V'' = (V')'$ denotes the causal completion of a convex spacetime region V.) Hence, for each bounded region V (convex combinations of) the states ϕ induced by analytic vectors are faithful on each such algebra $\mathcal{A}(V)$ (*i.e.* $X \in \mathcal{A}(V)$ and $\phi(XX^*) = 0$ imply $X = 0$). Such states are said to be locally faithful. We emphasize: given assumptions (i)–(v), all physically interesting states in the vacuum representation will be locally faithful.

We shall also assume that (vi) the net $\{\mathcal{A}(V)\}$ and state ϕ have a nontrivial scaling limit, either in the sense of Fredenhagen (1985) or in the sense of Buchholz and Verch (1996). This assumption has been verified in many concrete models and is expected to hold in any renormalizable quantum field theory with an ultraviolet fixed point, hence in all asymptotically free theories. The role of this physically motivated assumption in our argument is to provide information about the type of the local algebras $\mathcal{A}(V)$ which can occur.

Definition 1. The net $\{\mathcal{A}(V)\}$ is said to satisfy the *local primitive causality* condition if $\mathcal{A}(V'') = \mathcal{A}(V)$ for every nonempty convex region V.

Local primitive causality postulates that the quantum field undergoes a hyperbolic propagation within lightlike characteristics (Haag and Schroer, 1962). (See the discussion in Section 4 for further insight into the nature of this postulate.) Our final assumption is that the net satisfies local primitive causality. This assumption does not follow from assumptions (i)–(vi) (see Garber, 1975). However, this condition has been verified in many concrete models.

For spacetime point $x \in M$ let $V_+(x)$ ($V_-(x)$) denote the open forward (backward) light cones with apex x. If $x \in V_+(y)$ then $V_-(x) \cap V_+(y)$ is called a double cone. If V is a double cone, then V and V' are nonempty and $V = V''$. The wedge regions are Poincaré transforms of the basic wedge

$$W_R = \{(x_0, x_1, x_2, x_3) \in M \mid x_1 > |x_0|\}.$$

Note that wedges are unbounded sets and $W = W''$ for every wedge W. Moreover, if W is a wedge, then so is W'. As shown in Fredenhagen (1985), assumptions (i)–(vi) entail that the algebra $\mathcal{A}(V)$ is type III whenever V is a double cone or a wedge.

We shall need some definitions and results concerning the independence of local algebras.[4] A pair $(\mathcal{A}_1, \mathcal{A}_2)$ of C^*-subalgebras of the C^*-algebra $\mathcal{C}$ has the Schlieder property if $XY \neq 0$ for any $0 \neq X \in \mathcal{A}_1$ and $0 \neq Y \in \mathcal{A}_2$. Given assumptions (i)–(v), $(\mathcal{A}(V_1), \mathcal{A}(V_2))$ has the Schlieder property for all spacelike separated double cones or wedges (Summers, 1990).

A pair $(\mathcal{A}_1, \mathcal{A}_2)$ of such algebras is called C^*-independent if for any state ϕ_1 on $\mathcal{A}_1$ and for any state ϕ_2 on $\mathcal{A}_2$ there exists a state ϕ on $\mathcal{C}$ which extends both ϕ_1 and ϕ_2. Under assumptions (i)–(v), algebras associated with spacelike separated double cones are C^*-independent, since they form a mutually commuting pair of algebras satisfying the Schlieder property, which in this context is equivalent with C^*-independence (Roos, 1970).

Two von Neumann subalgebras $\mathcal{N}_1, \mathcal{N}_2$ of the von Neumann algebra $\mathcal{N}$ are called logically independent (Rédei, 1995a,b) if $A \wedge B \neq 0$ for any projections $0 \neq A \in \mathcal{N}_1$, $0 \neq B \in \mathcal{N}_2$. If $(\mathcal{N}_1, \mathcal{N}_2)$ is a mutually commuting pair, then C^*-independence and logical independence are equivalent (Rédei, 1998).[5] So we conclude:

Lemma 1. *Assumptions (i)–(v) entail that the pair* $(\mathcal{A}(V_1), \mathcal{A}(V_2))$ *is logically independent for any spacelike separated double cones or wedges* V_1, V_2.

Let V_1 and V_2 be two spacelike separated spacetime regions and $A \in \mathcal{A}(V_1)$ and $B \in \mathcal{A}(V_2)$ be two projections. If ϕ is a state on $\mathcal{A}(V_1 \cup V_2)$ and

$$\phi(A \wedge B) > \phi(A)\phi(B), \tag{1}$$

[4] For the origin and a detailed analysis of the interrelation of these and other notions of statistical independence, see the review (Summers 1990) and Chapter 11 in Rédei (1998)—for more recent results, see Florig and Summers (1997) and Hamhalter (1997).

[5] If $\mathcal{N}_1, \mathcal{N}_2$ do not mutually commute, then C^*-independence is strictly weaker than logical independence (Hamhalter, 1997).

then we say that there is *superluminal (or spacelike) correlation* between A and B in the state ϕ. We now explain why such correlations are common when assumptions (i)–(v) hold.

The ubiquitous presence of superluminal correlations is one of the consequences of the generic violation of Bell's inequalities in AQFT. To make this clear, recall (cf. Summers and Werner, 1985) that the Bell correlation $\beta(\phi, \mathcal{N}_1, \mathcal{N}_2)$ between two commuting von Neumann subalgebras $\mathcal{N}_1, \mathcal{N}_2$ of the von Neumann algebra $\mathcal{N}$ in state ϕ on $\mathcal{N}$ is defined by

$$\beta(\phi, \mathcal{N}_1, \mathcal{N}_2) \equiv \sup \frac{1}{2}\phi(X_1(Y_1 + Y_2) + X_2(Y_1 - Y_2)), \tag{2}$$

where the supremum in (2) is taken over all self–adjoint contractions $X_i \in \mathcal{N}_1, Y_j \in \mathcal{N}_2$. It can be shown (Cirel'son, 1980; Summers and Werner, 1987a) that $\beta(\phi, \mathcal{N}_1, \mathcal{N}_2) \leq \sqrt{2}$. The Clauser–Holt–Shimony–Horne version of Bell's inequality in this notation reads:

$$\beta(\phi, \mathcal{N}_1, \mathcal{N}_2) \leq 1, \tag{3}$$

and a state ϕ for which $\beta(\phi, \mathcal{N}_1, \mathcal{N}_2) > 1$ is called *Bell correlated.* It is known (Summers and Werner, 1987a) that if ϕ is a product state across the algebras $\mathcal{N}_1, \mathcal{N}_2$ (*i.e.*, if $\phi(XY) = \phi(X)\phi(Y)$, for all $X \in \mathcal{N}_1$ and $Y \in \mathcal{N}_2$), then $\beta(\phi, \mathcal{N}_1, \mathcal{N}_2) = 1$.

Lemma 2. *(Rédei and Summers, 2002) Let $\mathcal{N}_1$ and $\mathcal{N}_2$ be commuting subalgebras of the von Neumann algebra $\mathcal{N}$ and let ϕ be a normal state on $\mathcal{N}$ which is not a product state across the algebras $\mathcal{N}_1, \mathcal{N}_2$. Then there exist projections $A \in \mathcal{N}_1$ and $B \in \mathcal{N}_2$ such that $\phi(A \wedge B) > \phi(A)\phi(B)$.*

There are many situations in which $\beta(\phi, \mathcal{A}(V_1), \mathcal{A}(V_2)) = \sqrt{2}$ (cf. Summers and Werner, 1987a; Summers and Werner, 1987b; Summers and Werner, 1988). We recall a recent result by Halvorson and Clifton. Let the symbol $\mathcal{N}_1 \vee \mathcal{N}_2$ denote the smallest von Neumann algebra containing both $\mathcal{N}_1$ and $\mathcal{N}_2$.

Proposition 2. *(Halvorson and Clifton, 2000) If $(\mathcal{N}_1, \mathcal{N}_2)$ is a pair of commuting type III von Neumann algebras acting on the Hilbert space $\mathcal{H}$ and having the Schlieder property, then the set of unit vectors which induce Bell correlated states on $(\mathcal{N}_1, \mathcal{N}_2)$ is open and dense in the unit sphere of $\mathcal{H}$. Indeed, the set of normal states on $\mathcal{N}_1 \vee \mathcal{N}_2$ which are Bell correlated on $(\mathcal{N}_1, \mathcal{N}_2)$ is norm dense in the normal state space of $\mathcal{N}_1 \vee \mathcal{N}_2$.*

We see then that, given the assumptions (i)–(vi), for any spacelike separated double cones or wedges V_1, V_2, the pair $(\mathcal{A}(V_1), \mathcal{A}(V_2))$ satisfies the hypothesis of Prop. 2. So, "most" normal states on such pairs of algebras manifest superluminal correlations (1). Hence, superluminal correlations abound in AQFT, and the question posed in the introduction is not vacuous.

3. THE NOTION OF REICHENBACHIAN COMMON CAUSE IN AQFT

The following definition is a natural formulation in a noncommutative probability space $(\mathcal{P}(\mathcal{N}), \phi)$[6] of the classical notion of common cause given by Reichenbach (Reichenbach, 1956, Section 19).

Definition 2. Let $A, B \in \mathcal{P}(\mathcal{N})$ be two commuting projections which are correlated in ϕ:

$$\phi(A \wedge B) > \phi(A)\phi(B). \tag{4}$$

$C \in \mathcal{P}(\mathcal{N})$ is a *common cause* of the correlation (4) if C commutes with both A and B and the following conditions hold:

$$\phi(A \wedge B|C) = \phi(A|C)\phi(B|C), \tag{5}$$

$$\phi(A \wedge B|C^{\perp}) = \phi(A|C^{\perp})\phi(B|C^{\perp}), \tag{6}$$

$$\phi(A|C) > \phi(A|C^{\perp}), \tag{7}$$

$$\phi(B|C) > \phi(B|C^{\perp}). \tag{8}$$

($\phi(X|Y)$ denotes the conditional probability $\phi(X|Y) = \phi(X \wedge Y)/\phi(Y)$.)
For spacelike separated spacetime regions V_1 and V_2 let us define the following regions

$$wpast(V_1, V_2) \equiv (BLC(V_1) \setminus V_1) \cup (BLC(V_2) \setminus V_2), \tag{9}$$

$$cpast(V_1, V_2) \equiv (BLC(V_1) \setminus V_1) \cap (BLC(V_2) \setminus V_2), \tag{10}$$

$$spast(V_1, V_2) \equiv \cap_{x \in V_1 \cup V_2} BLC(x), \tag{11}$$

where $BLC(V)$ denotes the union of the backward lightcones of every point in V. Region $spast(V_1, V_2)$ consists of spacetime points *each* of which can causally influence *every* point in both V_1 and V_2; region $cpast(V_1, V_2)$ consists of spacetime points *each* of which can causally influence at least some point in *both* V_1 *and* V_2, and region $wpast(V_1, V_2)$ consists of spacetime points *each* of which can causally influence at least *some* point in *either* V_1 *or* V_2.

Definition 3. Let $\{\mathcal{A}(V)\}$ be a net of local von Neumann algebras over Minkowski space. Let V_1 and V_2 be two spacelike separated spacetime regions, and let ϕ be

[6] $\mathcal{P}(\mathcal{N})$ is the set of all projections in the von Neumann algebra $\mathcal{N}$.

a locally normal state on the net. If for any pair of projections $A \in \mathcal{A}(V_1)$ and $B \in \mathcal{A}(V_2)$ the inequality

$$\phi(A \wedge B) > \phi(A)\phi(B) \tag{12}$$

entails the existence of a projection C in the von Neumann algebra $\mathcal{A}(V)$ which is a common cause of the correlation (12) in the sense of Definition 2, then the local system $(\mathcal{A}(V_1), \mathcal{A}(V_2), \phi)$ is said to satisfy the

$$\textbf{Weak Common Cause Principle} \text{ if } \quad V \subseteq wpast(V_1, V_2), \tag{13}$$

$$\textbf{Common Cause Principle} \text{ if } \quad V \subseteq cpast(V_1, V_2), \tag{14}$$

$$\textbf{Strong Common Cause Principle} \text{ if } \quad V \subseteq spast(V_1, V_2). \tag{15}$$

We say that Reichenbach's Common Cause Principle holds for the net (respectively holds in the weak or strong sense) iff for every pair of spacelike separated convex spacetime regions V_1, V_2 and every normal state ϕ, the Common Cause Principle holds for the local system $(\mathcal{A}(V_1), \mathcal{A}(V_2), \phi)$ (respectively in the weak or strong sense).

If V_1 and V_2 are complementary wedges then $spast(V_1, V_2) = \emptyset$. Since the local von Neumann algebras pertaining to complementary wedges are known to contain correlated projections (see Summers and Werner, 1988 and Summers, 1990), the *Strong* Reichenbach's Common Cause Principle trivially fails in AQFT.

The problem of whether the Common Cause Principle holds in AQFT was raised in Rédei (1997), and the problem is still open. For the Weak Common Cause Principle we have the following result.

Proposition 3. *If the net $\{\mathcal{A}(V)\}$ satisfies conditions (i)–(vi) and local primitive causality, then every local system $(\mathcal{A}(V_1), \mathcal{A}(V_2), \phi)$ with V_1, V_2 nonempty convex open sets such that V_1'' and V_2'' are spacelike separated double cones and with a locally normal and locally faithful state ϕ satisfies the Weak Common Cause Principle.*

The proof of Proposition 3 is based on the following two lemmas:

Lemma 3. *Let ϕ be a faithful state on a von Neumann algebra $\mathcal{N}$ containing two mutually commuting subalgebras $\mathcal{N}_1, \mathcal{N}_2$ which are logically independent. Let $A \in \mathcal{N}_1$ and $B \in \mathcal{N}_2$ be projections satisfying (4). Then a sufficient condition for C to satisfy (5)–(8) is that the following two conditions hold:*

$$C < A \wedge B, \tag{16}$$

$$\phi(C) = \frac{\phi(A \wedge B) - \phi(A)\phi(B)}{1 - \phi(A \vee B)}. \tag{17}$$

Lemma 4. *Let $\mathcal{N}$ be a type III von Neumann algebra on a separable Hilbert space $\mathcal{H}$, and let ϕ be a faithful normal state on $\mathcal{N}$. Then for every projection $A \in \mathcal{P}(\mathcal{N})$ and every positive real number $0 < r < \phi(A)$ there exists a projection $P \in \mathcal{P}(\mathcal{N})$ such that $P < A$ and $\phi(P) = r$.*

Due to space limitations, we must refer the reader to Rédei and Summers (2002) for the proof of these assertions.

4. FINAL REMARKS

The local primitive causality condition plays an essential role in our proof of Prop. 3 but is barely discussed in the literature. If V is a convex region, then for any point $x \in V''$, every inextendible causal curve through x must intersect V. Hence, the values of a classical quantum field satisfying a hyperbolic equation of motion whose speed of propagation is bounded by that of light would at every point in V'' be completely determined by its values in V. This well-known state of affairs finds an analogous expression in quantum field theory in the condition $\mathcal{A}(V) = \mathcal{A}(V'')$. For free quantum fields there is an explicit link between the mentioned fact about classical fields and the condition $\mathcal{A}(V) = \mathcal{A}(V'')$—cf. Glimm and Jaffe (1972). For interacting quantum fields, the link is significantly more indirect, but has been verified in many concrete models—see again (Glimm and Jaffe, 1972) for references. For this reason, workers in AQFT take the condition of local primitive causality in general as an expression of hyperbolic propagation within lightlike characteristics.

The validity of the local primitive causality condition leads to some consequences which are nonintuitive to many who are first exposed to its uses. In particular, since it is clearly possible for two disjoint regions V_1, V_2 to be contained in the casual completion V'' of a third region V, itself disjoint from $V_1 \cup V_2$, it is possible for a single element $A \in \mathcal{A}(V'')$ to be an element of both $\mathcal{A}(V_1 \cup V_2)$ and $\mathcal{A}(V)$ and therefore to be localized in mutually disjoint regions.[7] Since the operational interpretation of a self-adjoint $A \in \mathcal{A}(V)$ is that of an observable measurable in V, this leads to some initial conceptual discomfort.

This discomfort is dissolved by noting that an "observable" A does not represent a unique measuring apparatus in some fixed laboratory, but rather represents an equivalence class of such apparata (cf. Neumann and Werner, 1983). Consider two such idealized apparata X, Y such that $\phi(X) = \phi(Y)$ for all (idealized) states ϕ admitted in the theory (the set of such states contains as a subset—at least in principle—all states preparable in the laboratory). These two apparata are then identified to be in the same equivalence class and are thus represented by a single operator A. Hence, the element A above, which is localized simultaneously in V and $V_1 \cup V_2$, represents two distinct events—one taking place in V and the other taking place in $V_1 \cup V_2$. The fact that it is possible, given any event in $V_1 \cup V_2$, to find an event in V which is equivalent to the first in the stated sense is part of

[7] Indeed, this fact is essential in our proof of Prop. 3.

the content of the local primitive causality condition. It is therefore of interest that one can actually verify this condition in models.

Of further relevance to our purposes is the observation that the use of local primitive causality leads to the conclusion that two correlated projections A, B yield an infinity of events, each of which is localized in a manner disjoint from the others and is a strong common cause of A and B in the sense of (16). Relativistic quantum field theory is extremely rich in such strong common causes!

Proposition 3 locates the common cause C within the union of the backward light cones of V_1 and V_2; however, a bit more can be said of its location. Define $\tilde{V}_1$ and $\tilde{V}_2$ by

$$\tilde{V}_1 \equiv (BLC(V_1) \cap V) \setminus (BLC(V_1) \cap BLC(V_2)) \tag{18}$$

$$\tilde{V}_2 \equiv (BLC(V_2) \cap V) \setminus (BLC(V_1) \cap BLC(V_2)) \tag{19}$$

Since $(\tilde{V}_1 \cup V_1)$ and $(\tilde{V}_2 \cup V_2)$ are contained in spacelike separated double cones, the algebras $\mathcal{N}(\tilde{V}_1 \cup V_1)$ and $\mathcal{N}(\tilde{V}_2 \cup V_2)$ are logically independent, hence the common cause $C < A \wedge B$ cannot belong to $\mathcal{N}(\tilde{V}_1)$ or to $\mathcal{N}(\tilde{V}_2)$ only, so neither $V \subseteq \tilde{V}_1$ nor $V \subseteq \tilde{V}_2$ is possible.

Finally, we note that the existence of a common cause in the presence of a violation of Bell's inequalities may seem paradoxical, because the violation of Bell's inequalities is represented by some (see, *e.g*, van Fraassen, 1982) as implying the nonexistence of a common cause. But there is no contradiction here—it is essential to realize (cf. Rédei, 1997; Hofer-Szabó et al., 1999) that Bell's inequality involves four pairs of correlated projections. To show that Bell's inequality must hold, van Fraassen (1982) effectively assumes that all pairs have the same common cause, *i.e.* a *common* common cause C. We have demonstrated that a given pair of correlated projections has a common cause, not that some set of four correlated pairs has a common common cause. Common common causes for different correlations do not exist in general even in classical probability theory, as shown in Hofer-Szabó et al. (2002).

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REFERENCES

Borchers, H.-J. (1965). On the vacuum state in quantum field theory, II. *Communications in Mathematical Physics* **1**, 57–79.

Buchholz, D. and Verch, R. (1996). Scaling algebras and renormalization group in algebraic quantum field theory. *Reviews in Mathematical Physics* **7**, 1195–1239.

Cirel'son, B. S. (1980). Quantum generalizations of Bell's inequalities, *Letters in Mathematical Physics* **4**, 93–100.

Florig, M. and Summers, S. J. (1997). On the statistical independence of algebras of observables, *Journal of Mathematical Physics* **38**, 1318–1328.

Fredenhagen, K. (1985). On the modular structure of local algebras of observables, *Communications in Mathematical Physics* **97**, 79–89.

Garber, W. D. (1975). The connexion of duality and causality properties for generalized free fields, *Communications in Mathematical Physics* **42**, 195–208.

Glimm, J. and Jaffe, A. (1972). Boson quantum field models. In *Mathematics in Contemporary Physics*, Streater, R. F., ed., Academic Press, New York.

Haag, R. and Schroer, B. (1962). Postulates of quantum field theory, *Journal of Mathematical Physics* **3**, 248–256.

Haag, R. (1992), *Local Quantum Physics*, Springer Verlag, Berlin.

Halvorson, H. and Clifton, R. (2000). Generic Bell correlation between arbitrary local algebras in quantum field theory, *Journal of Mathematical Physics* **41**, 1711–1717.

Hamhalter, J. (1997). Statistical independence of operator algebras, *Annales de l'Institut Henri Poincaré - Physique théorique* **67**, 447–462.

Hofer-Szabó, G., Rédei, M., and Szabó, L. E. (1999). On Reichenbach's Common Cause Principle and Reichenbach's notion of common cause, *British Journal for the Philosophy of Science* **50**, 377–399.

Hofer-Szabó, G., Rédei, M., and Szabó, L. E. (2002). Common-Causes are not common common-causes, *Philosophy of Science* **69**, 623–636.

Horuzhy, S. S. (1990), *Introduction to Algebraic Quantum Field Theory*, Kluwer Academic Publishers, Dordrecht.

Neumann, H. and Werner, R. (1983). Causality between preparation and registration processes in relativistic quantum theory, *International Journal of Theoretical Physics* **22**, 781–802.

Rédei, M. (1995a). Logically independent von Neumann lattices, *International Journal of Theoretical Physics* **34**, 1711–1718.

Rédei, M. (1995). Logical independence in quantum logic, *Foundations of Physics* **25**, 411–422.

Rédei, M. (1997). Reichenbach's common cause principle and quantum field theory, *Foundations of Physics* **27**, 1309–1321.

Rédei, M. (1998), *Quantum Logic in Algebraic Approach*, Kluwer Academic Publishers, Dordrecht.

Rédei, M. and Summers, S. J. (2002). Local primitive causality and the Common Cause Principle in quantum field theory, *Foundations of Physics* **32**, 335–355.

Reichenbach, H. (1956), *The Direction of Time*, University of California Press, Los Angeles.

Roos, H.-J. (1970). Independence of local algebras in quantum field theory, *Communications in Mathematical Physics* **16**, 238–246.

Salmon, W. C. (1984) *Scientific Explanation and the Causal Structure of the World*, Princeton University Press, Princeton.

Summers, S. J. (1990). On the independence of local algebras in quantum field theory, *Reviews in Mathematical Physics* **2**, 201–247.

Summers, S. J. and Werner, R. (1985). The vacuum violates Bell's inequalities, *Physics Letters A* **110**, 257–279.

Summers, S. J. and Werner, R. (1987a). Bell's inequalities and quantum field theory, I, General setting, *Journal of Mathematical Physics* **28**, 2440–2447.

Summers, S. J. and Werner, R. (1987b). Maximal violation of Bell's inequalities is generic in quantum field theory, *Communications in Mathematical Physics* **110**, 247–259.

Summers, S. J. and Werner, R. (1988). Maximal violation of Bell's inequalities for algebras of observables in tangent spacetime regions, *Annales de l'Institut Henri Poincaré - Physique théorique* **49**, 215–243.

van Fraassen, B. C. (1982). The Charybdis of realism: Epistemological implications of Bell's inequalities, *Synthese* **52**, 25–38.